

Exercise 66

Suppose g is an odd function and let $h = f \circ g$. Is h always an odd function? What if f is odd? What if f is even?

Solution

h is defined as the composition of f and g .

$$\begin{aligned}h(x) &= f \circ g \\ &= f(g(x))\end{aligned}$$

Check to see if $h(x)$ is an odd function, noting that g is an odd function: $g(-x) = -g(x)$.

$$\begin{aligned}h(-x) &= f(g(-x)) \\ &= f(-g(x))\end{aligned}$$

h is not necessarily an odd function. If f is also odd, then h is odd.

$$h(-x) = -f(g(x)) = -h(x)$$

But if f is even, then h is even.

$$h(-x) = f(g(x)) = h(x)$$